

Lecture 13 - Parametric NLP

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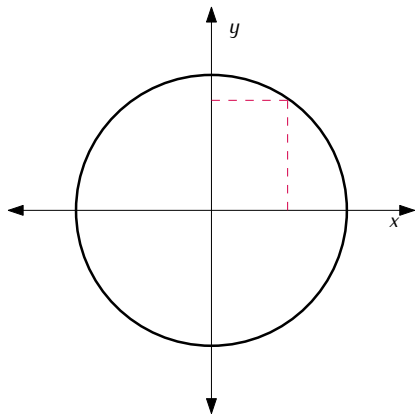
Spring 2026

Sensitivites of implicit functions

- Consider a function given by

$$x^2 + y^2 = 1$$

- Here y is not given as an explicit function of x
- But for certain values of x , there exists corresponding values of y
- Express this as $F(x, y(x)) = 0$



How does y change, when x changes?

Sensitivites of implicit functions

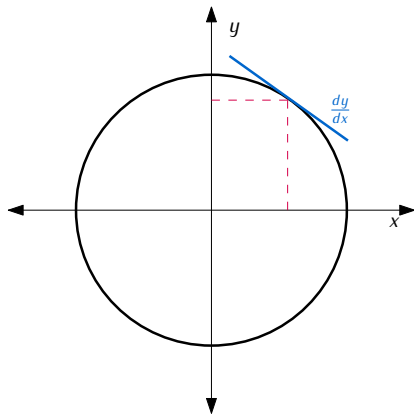
Implicit Function Theorem

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \frac{\partial y}{\partial x} = 0$$

$$\frac{d}{dx}(x^2 + y^2 - 1) = 0$$

$$2x + 2y \frac{\partial y}{\partial x} = 0$$

$$\frac{\partial y}{\partial x} = -\frac{x}{y}$$



Implicit Function Theorem

For a function $F(x, y(x))$,

- ▶ if there is a solution to $F(x, y) = 0$ around some point (x_0, y_0)
- ▶ if $\exists \frac{\partial F}{\partial y} \neq 0$ at (x_0, y_0)

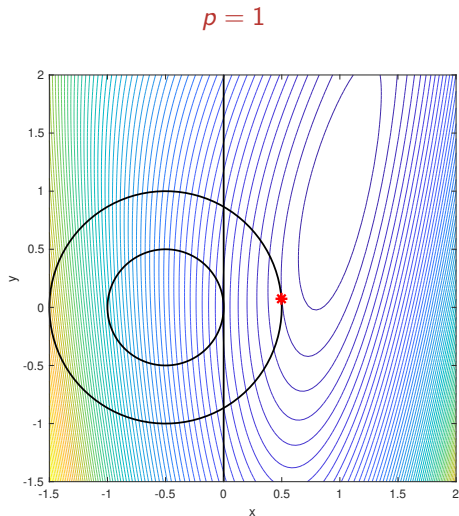
then there exists a unique differentiable function $y(x)$ near x_0 such that $F(x, y(x)) = 0$.

Applying the chain rule to differentiate $F(x, y(x)) = 0$ with respect to x :

$$\frac{d}{dx} F(x, y(x)) = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \frac{\partial y}{\partial x} = 0.$$

Parametric NLP

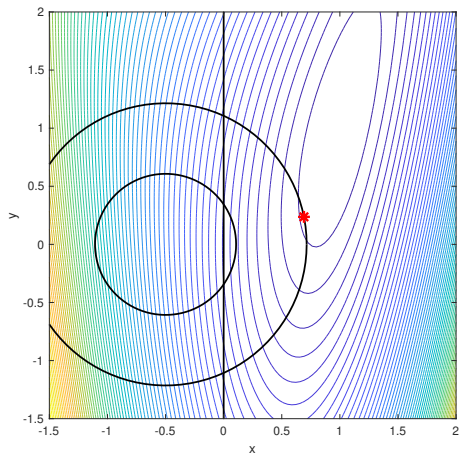
$$\begin{aligned} \min_x \quad & (1 - w_1)^2 + 0.2(w_2 - w_1^2)^2 \\ \text{s.t.} \quad & w_1 \geq 0, \\ & \left(\frac{p}{2}\right)^2 \leq (w_1 + 0.5)^2 + w_2^2 \leq p^2. \end{aligned}$$



Parametric NLP

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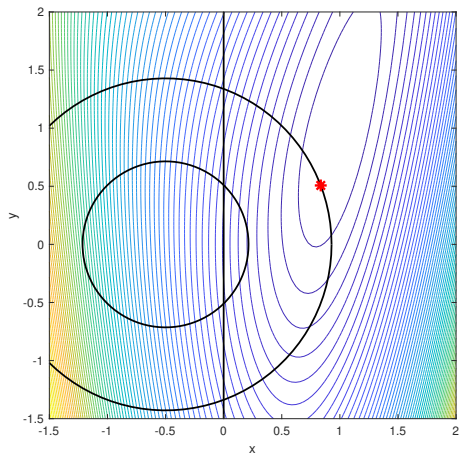
$$p = 1.2143$$



Parametric NLP

$$\begin{aligned} \min_x \quad & (1 - w_1)^2 + 0.2(w_2 - w_1^2)^2 \\ \text{s.t.} \quad & w_1 \geq 0, \\ & \left(\frac{p}{2}\right)^2 \leq (w_1 + 0.5)^2 + w_2^2 \leq p^2. \end{aligned}$$

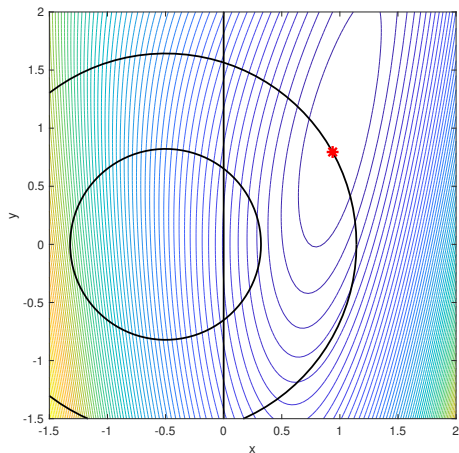
$$p = 1.4286$$



Parametric NLP

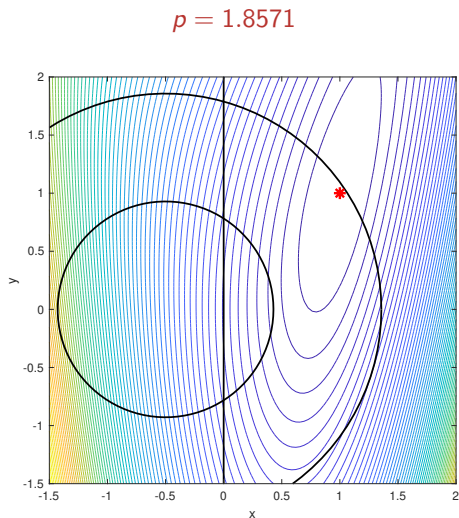
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$$p = 1.6429$$



Parametric NLP

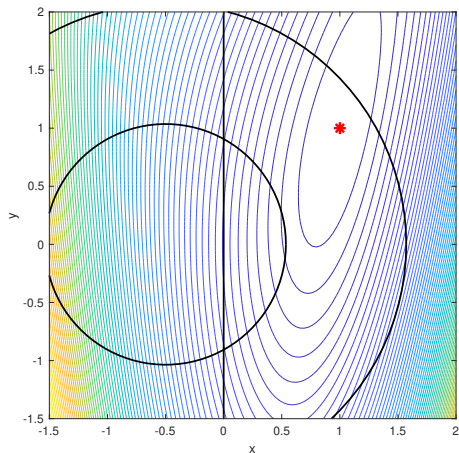
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Parametric NLP

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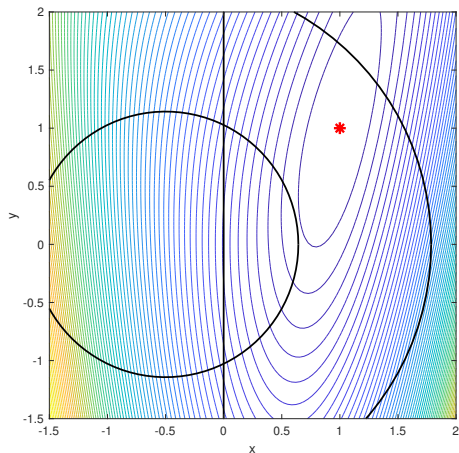
$$p = 2.0714$$



Parametric NLP

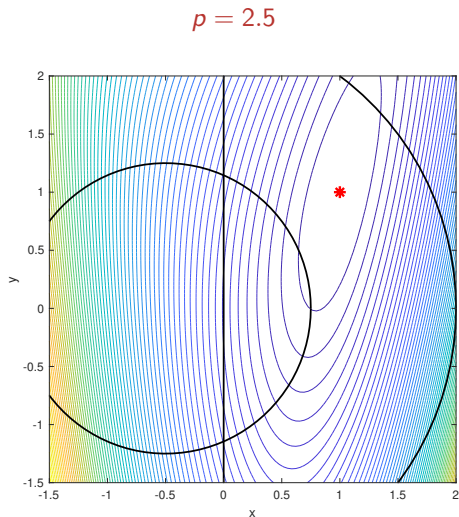
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$$p = 2.2857$$

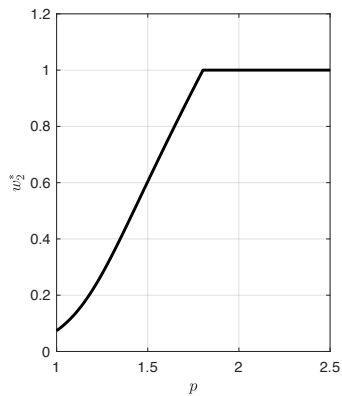
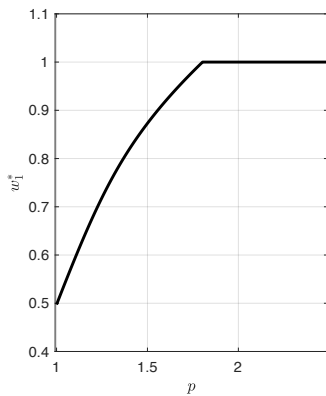


Parametric NLP

$$\begin{aligned} \min_x \quad & (1 - w_1)^2 + 0.2(w_2 - w_1^2)^2 \\ \text{s.t.} \quad & w_1 \geq 0, \\ & \left(\frac{p}{2}\right)^2 \leq (w_1 + 0.5)^2 + w_2^2 \leq p^2. \end{aligned}$$



Parametric NLP



Parametric NLP

Consider the parametric NLP problem, where we want to solve the problem for a given “exogenous” parameter $p \in \mathbb{R}^{n_p}$

$$\begin{aligned} \min_x \quad & f(x, p) \\ \text{s.t.} \quad & h(x, p) = 0 \\ & g(x, p) \leq 0 \end{aligned}$$

Parametric NLP

Consider the parametric NLP problem, where we want to solve the problem for a given “exogenous” parameter $p \in \mathbb{R}^{n_p}$

$$\begin{aligned} \min_x \quad & f(x, p) \\ \text{s.t.} \quad & h(x, p) = 0 \\ & g(x, p) \leq 0 \end{aligned}$$

Solution x^* is an implicit function of parameters p , i.e.,
 $x^*(p) : \mathbb{R}^{n_w} \rightarrow \mathbb{R}^{n_p}$.

We would like to know...

- ▶ Is $x^*(p)$ continuous and differentiable?
- ▶ What does $x^*(p)$ tell us about $x^*(p + \Delta p)$
- ▶ How to keep track of $x^*(p)$ for continuously changing p ?

Parametric NLP - KKT conditions

$$\mathcal{L}(x, \lambda, \mu, p) = f(x, p) + \lambda^T h(x, p) + \mu^T g(x, p)$$

- The KKT conditions can be expressed as:

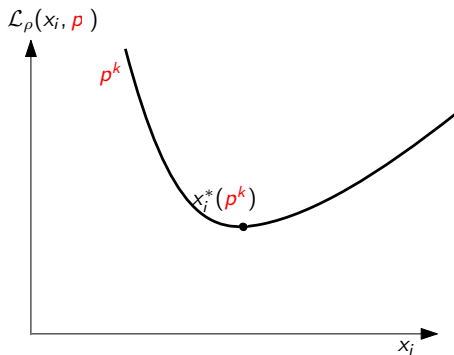
$$\varphi(s(p), p) := \begin{bmatrix} \nabla \mathcal{L}(x, \lambda, \mu, p) \\ h(x, p) \\ g_{\mathbb{A}}(x, p) \end{bmatrix} = 0$$
$$\mu_{\mathbb{A}} > 0, \mu_{\bar{\mathbb{A}}} = 0, g_{\bar{\mathbb{A}}}(x, p) < 0$$

- Primal-dual solution vector $s^*(p) := [x^*, \lambda^*, \mu^*]^T$.

Is $x^*(p)$ continuous and differentiable?

Assume

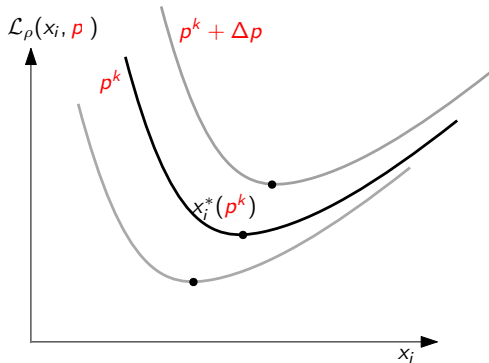
LICQ, SOSC, and strict complementarity



Is $x^*(p)$ continuous and differentiable?

Assume

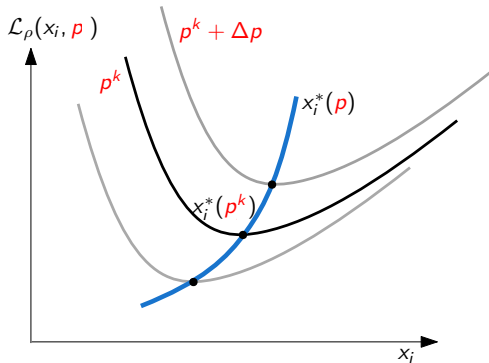
LICQ, SOSC, and strict complementarity



Is $x^*(p)$ continuous and differentiable?

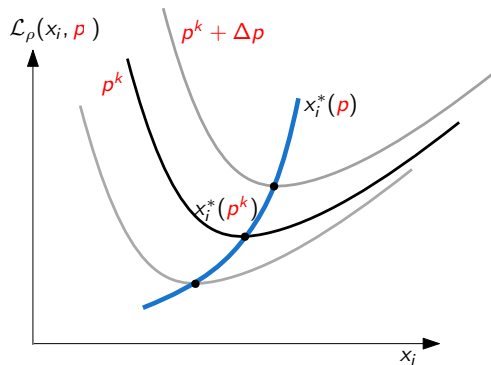
Assume

LICQ, SOSC, and strict complementarity

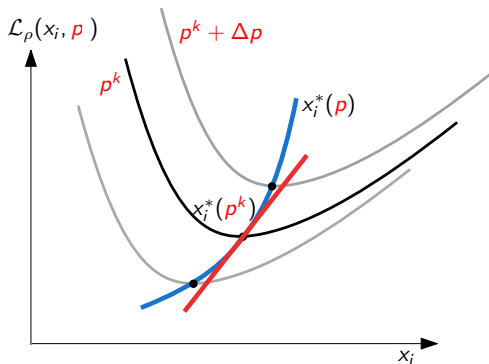


$\exists$ a continuous and differentiable solution manifold $s^*(p)$ that is an implicit function of p .

What does $x^*(p)$ tell us about $x^*(p + \Delta p)$



What does $x^*(p)$ tell us about $x^*(p + \Delta p)$



$$s^*(p^{k+1}) = s^*(p^k) + \frac{\partial s^*}{\partial p}(p^{k+1} - p^k) + \mathcal{O}\|\Delta p\|^2$$

What does $x^*(p)$ tell us about $x^*(p + \Delta p)$

Computing sensitivities

Implicit function theorem on the KKT conditions: $\varphi(s(p), p)$

$$\left. \frac{\partial \varphi}{\partial \mathbf{p}}(s(p), p) \right|_{p=p^k} = \underbrace{\frac{\partial \varphi}{\partial s} \frac{\partial s^*}{\partial \mathbf{p}}}_{:= \mathcal{M}} + \underbrace{\frac{\partial \varphi}{\partial \mathbf{p}}}_{:= \mathcal{N}} = 0$$

$$\frac{\partial s^*}{\partial \mathbf{p}} = -\mathcal{M}^{-1} \mathcal{N}$$

$$s^*(p^{k+1}) = s^*(p^k) + \frac{\partial s^*}{\partial \mathbf{p}}(p^{k+1} - p^k) + \mathcal{O} \|\Delta \mathbf{p}\|^2$$

What does $x^*(p)$ tell us about $x^*(p + \Delta p)$

$$\left. \frac{\partial \varphi}{\partial \mathbf{p}}(s(p), p) \right|_{p=p^k} = \underbrace{\frac{\partial \varphi}{\partial \mathbf{s}}}_{:=\mathcal{M}} \frac{\partial s^*}{\partial \mathbf{p}} + \underbrace{\frac{\partial \varphi}{\partial \mathbf{p}}}_{:=\mathcal{N}} = 0$$

$$\varphi(s(p), p) := \begin{bmatrix} \nabla \mathcal{L}(x, \lambda, \mu, p) \\ h(x, p) \\ g_{\mathbb{A}}(x, p) \end{bmatrix} = 0$$

$$\mathcal{M} := \frac{\partial \varphi}{\partial \mathbf{s}} = \begin{bmatrix} \nabla_{xx}^2 \mathcal{L}_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \quad \mathcal{N} := \frac{\partial \varphi}{\partial \mathbf{p}} = \begin{bmatrix} \nabla_{xp}^2 \mathcal{L}_k \\ \nabla_p h_k \\ \nabla_p g_{\mathbb{A},k} \end{bmatrix}$$

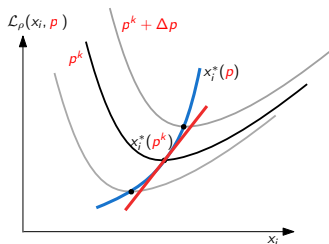
Tangential predictor / Linear predictor

$$s^*(p^{k+1}) \approx \hat{s}^*(p^{k+1}) = s^*(p^k) - \mathcal{M}^{-1} \mathcal{N}(p^{k+1} - p^k)$$

What does $x^*(p)$ tell us about $x^*(p + \Delta p)$

Linear predictor

$$\hat{s}^*(p^{k+1}) = s^*(p^k) - \mathcal{M}^{-1} \mathcal{N}(p^{k+1} - p^k)$$

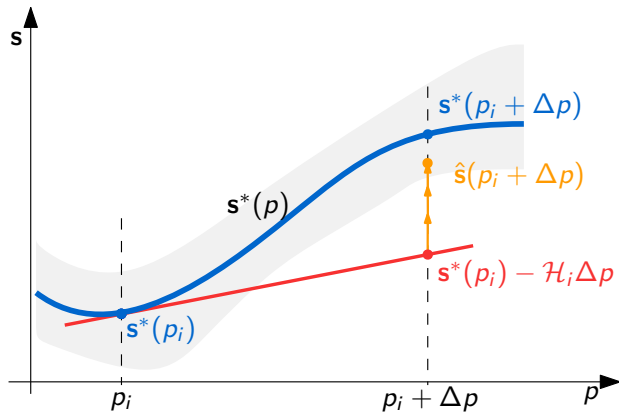


Approximation error

From continuity and differentiability of the optimal solution vector
 $\exists L_s > 0$ such that

$$\|\hat{s}^*(p^{k+1}) - s^*(p^{k+1})\| \leq L_s \|p^{k+1} - p^k\|^2$$

Path-following: How to keep track of $s^*(p)$?



Predictor-Corrector steps

► KKT conditions

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \lambda^* \\ \mu_{\mathbb{A}}^* \end{bmatrix} + \begin{bmatrix} \nabla_x \Phi \\ h \\ g_{\mathbb{A}} \end{bmatrix} = 0$$

Predictor-Corrector steps

► KKT conditions

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \lambda^* \\ \mu_{\mathbb{A}}^* \end{bmatrix} + \begin{bmatrix} \nabla_x \Phi \\ h \\ g_{\mathbb{A}} \end{bmatrix} = 0$$

► Linear Predictor

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \\ \Delta \mu_{\mathbb{A}} \end{bmatrix} + \begin{bmatrix} \nabla_{xp}^2 \mathcal{L}_k \\ \nabla_p h_k \\ \nabla_p g_{\mathbb{A},k} \end{bmatrix} \Delta p = 0$$

Predictor-Corrector steps

► KKT conditions

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \lambda^* \\ \mu_{\mathbb{A}}^* \end{bmatrix} + \begin{bmatrix} \nabla_x \Phi \\ h \\ g_{\mathbb{A}} \end{bmatrix} = 0$$

► Linear Predictor

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \\ \Delta \mu_{\mathbb{A}} \end{bmatrix} + \begin{bmatrix} \nabla_{xp}^2 \mathcal{L}_k \\ \nabla_p h_k \\ \nabla_p g_{\mathbb{A},k} \end{bmatrix} \Delta p = 0$$

► Sum $\Rightarrow$ Predictor-Corrector

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \lambda^* + \Delta \lambda \\ \mu_{\mathbb{A}}^* + \Delta \mu_{\mathbb{A}} \end{bmatrix} + \begin{bmatrix} \nabla_{xp}^2 \mathcal{L}_k \\ \nabla_p h_k \\ \nabla_p g_{\mathbb{A},k} \end{bmatrix} \Delta p + \begin{bmatrix} \nabla_x \Phi \\ h \\ g_{\mathbb{A}} \end{bmatrix} = 0$$

Predictor-Corrector steps

- KKT conditions

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \lambda^* \\ \mu_{\mathbb{A}}^* \end{bmatrix} + \begin{bmatrix} \nabla_x \Phi \\ h \\ g_{\mathbb{A}} \end{bmatrix} = 0$$

- Linear Predictor

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \\ \Delta \mu_{\mathbb{A}} \end{bmatrix} + \begin{bmatrix} \nabla_{xp}^2 \mathcal{L}_k \\ \nabla_p h_k \\ \nabla_p g_{\mathbb{A},k} \end{bmatrix} \Delta p = 0$$

- Sum $\Rightarrow$ Predictor-Corrector

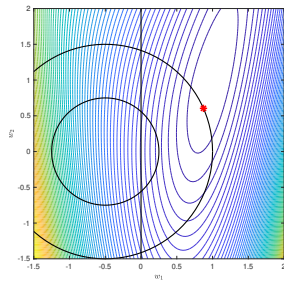
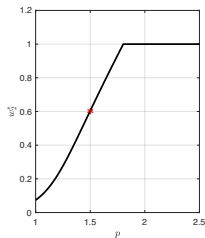
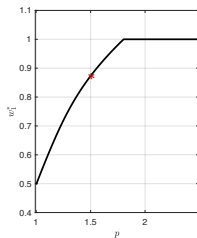
$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \lambda^* + \Delta \lambda \\ \mu_{\mathbb{A}}^* + \Delta \mu_{\mathbb{A}} \end{bmatrix} + \begin{bmatrix} \nabla_{xp}^2 \mathcal{L}_k \\ \nabla_p h_k \\ \nabla_p g_{\mathbb{A},k} \end{bmatrix} \Delta p + \begin{bmatrix} \nabla_x \Phi \\ h \\ g_{\mathbb{A}} \end{bmatrix} = 0$$

- Set $\Delta p = 0 \Rightarrow$ Pure Corrector

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \lambda^* + \Delta \lambda \\ \mu_{\mathbb{A}}^* + \Delta \mu_{\mathbb{A}} \end{bmatrix} + \begin{bmatrix} \nabla_x \Phi \\ h \\ g_{\mathbb{A}} \end{bmatrix} = 0$$

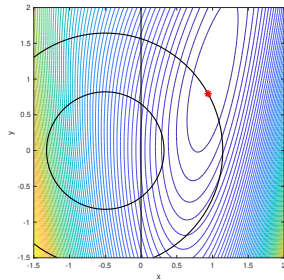
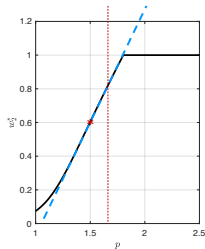
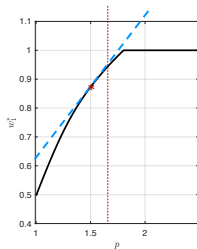
Parametric NLP Example

Solve NLP at $p = 1.5$



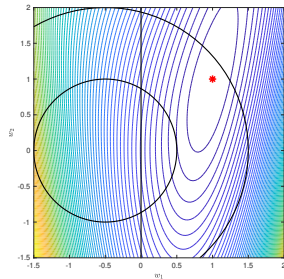
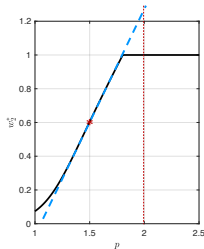
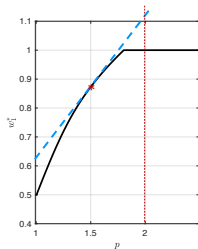
Parametric NLP Example

Approximate solution at $p = 1.643$



Parametric NLP Example

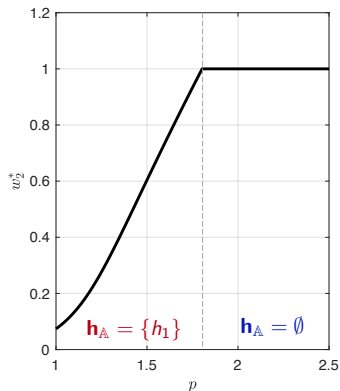
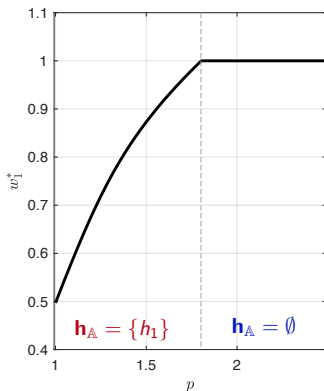
Approximate solution at $p = 2$



Active constraint set g_A changes!

Parametric NLP Example

Changes in the active constraint set $g_{\mathbb{A}}$ introduces non-smooth kinks in the solution manifold!



Non-smooth linear predictor

$$\begin{bmatrix} H_k & \nabla_x h_k & \nabla_x g_{\mathbb{A},k} \\ \nabla_x h_k^\top & 0 & 0 \\ \nabla_x g_{\mathbb{A},k}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \lambda^* + \Delta \lambda \\ \mu_{\mathbb{A}}^* + \Delta \mu_{\mathbb{A}} \end{bmatrix} + \begin{bmatrix} \nabla_{xp}^2 \mathcal{L}_k \\ \nabla_p h_k \\ \nabla_p g_{\mathbb{A},k} \end{bmatrix} \Delta p + \begin{bmatrix} \nabla_x \Phi \\ h \\ g_{\mathbb{A}} \end{bmatrix} = 0$$

is the directional derivative of the following QP

$$\begin{aligned} \min_{\Delta x} \quad & \frac{1}{2} \Delta x^\top H_k \Delta x + \Delta p^\top \nabla_{xp} \mathcal{L}_k \Delta x + \nabla_x f_k^\top \Delta x \\ \text{s.t.} \quad & \nabla_x g_k^\top \Delta x + \nabla_p g_k^\top \Delta p + g_k = 0 \quad | \quad \lambda_{QP} \\ & \nabla_x h_k^\top \Delta x + \nabla_p h_k^\top \Delta p + h_k \leq 0 \quad | \quad \mu_{QP} \end{aligned}$$

with $\lambda_{QP} = \lambda^* + \Delta \lambda$ and $\mu_{QP} = \mu^* + \Delta \mu$.

Non-smooth linear predictor

QP Predictor

$$\begin{aligned} \min_{\Delta x} \quad & \frac{1}{2} \Delta x^\top H_k \Delta x + \Delta p^\top \nabla_{xp} \mathcal{L}_k \Delta x \\ \text{s.t.} \quad & \nabla_x g_k^\top \Delta x + \nabla_p g_k^\top \Delta p + g_k = 0 \\ & \nabla_x h_k^\top \Delta x + \nabla_p h_k^\top \Delta p + h_k \leq 0 \end{aligned}$$

SQP Corrector

$$\begin{aligned} \min_{\Delta x} \quad & \frac{1}{2} \Delta x^\top H_k \Delta x + \nabla_x f_k^\top \Delta x \\ \text{s.t.} \quad & \nabla_x g_k^\top \Delta x + g_k = 0 \\ & \nabla_x h_k^\top \Delta x + h_k \leq 0 \end{aligned}$$

Non-smooth linear predictor

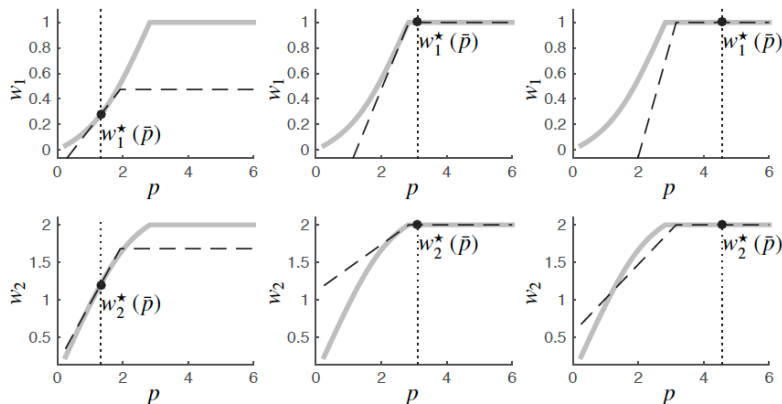


Figure Source: Diehl and Gros, Numerical Optimal Control

Predictor corrector methods for path-following

QP Predictor

$$\begin{aligned} \min_{\Delta x} \quad & \frac{1}{2} \Delta x^\top H_k \Delta x + \Delta p^\top \nabla_{xp} \mathcal{L}_k \Delta x \\ \text{s.t.} \quad & \nabla_x g_k^\top \Delta x + \nabla_p g_k^\top \Delta p + g_k = 0 \\ & \nabla_x h_k^\top \Delta x + \nabla_p h_k^\top \Delta p + h_k \leq 0 \end{aligned}$$

SQP Corrector

$$\begin{aligned} \min_{\Delta x} \quad & \frac{1}{2} \Delta x^\top H_k \Delta x + \nabla_x f_k^\top \Delta x \\ \text{s.t.} \quad & \nabla_x g_k^\top \Delta x + g_k = 0 \\ & \nabla_x h_k^\top \Delta x + h_k \leq 0 \end{aligned}$$

Predictor-Corrector path following

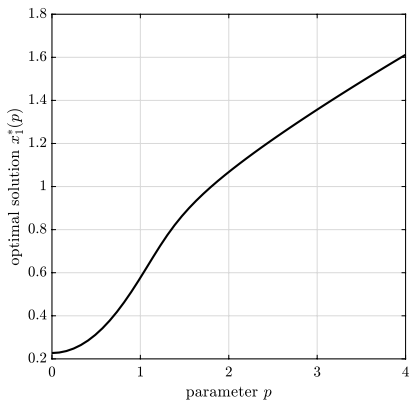
Solution $s^*(p^0)$ known at p^0 , together with $H, h_k, \nabla h_k, \dots$

For $k = 0, 1, 2, \dots$

1. Solve predictor with $\Delta p = p^{k+1} - p^k$
2. Correct solution until KKT fulfilled
3. Re-evaluate $H, h_k, \nabla h_k, \dots$ at new solution, and go to 1.

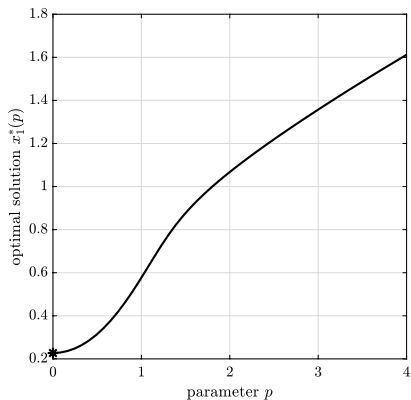
3. Path-following: How to keep track of $\mathbf{s}^*(p)$?

Optimal solution manifold for an NLP



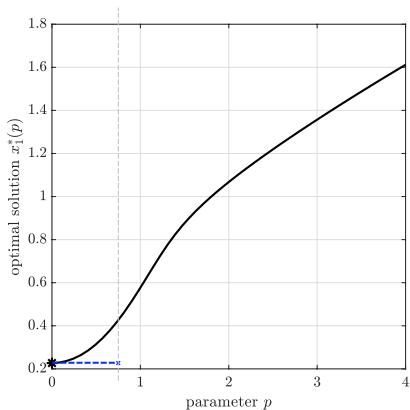
3. Path-following: How to keep track of $\mathbf{s}^*(p)$?

Solve NLP at $p = 0$



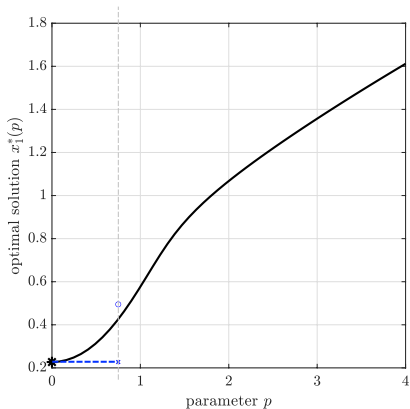
3. Path-following: How to keep track of $\mathbf{s}^*(p)$?

Tangential predictor



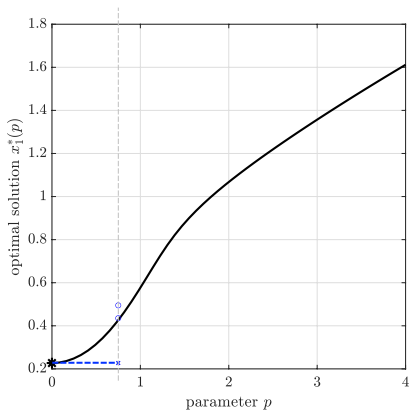
3. Path-following: How to keep track of $\mathbf{s}^*(p)$?

Enforce KKT conditions using additional **corrector** steps until $\|\nabla \mathcal{L}_{\rho,i}(\tilde{x}_i^{k+1}, p_i^{k+1})\| \leq D$



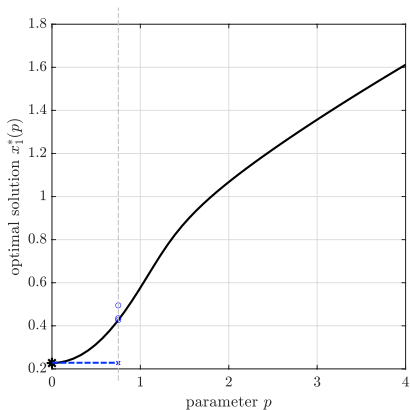
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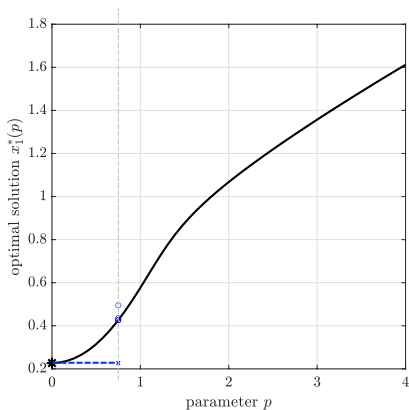
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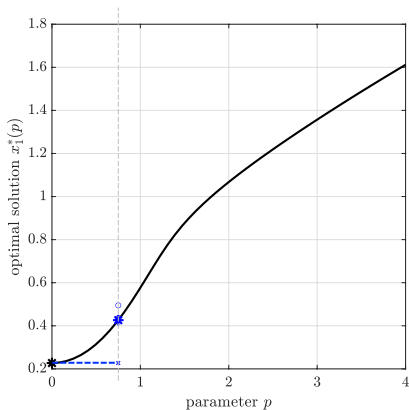
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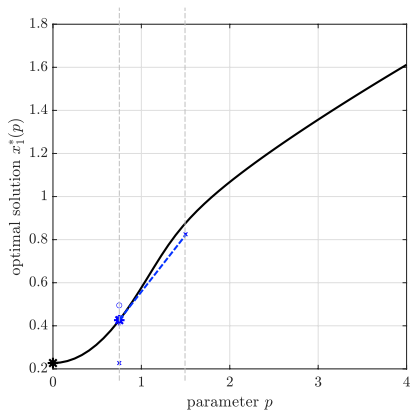
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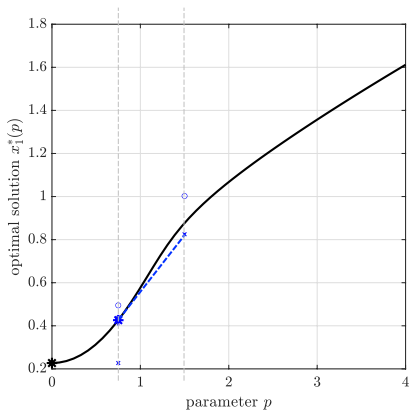
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Tangential predictor



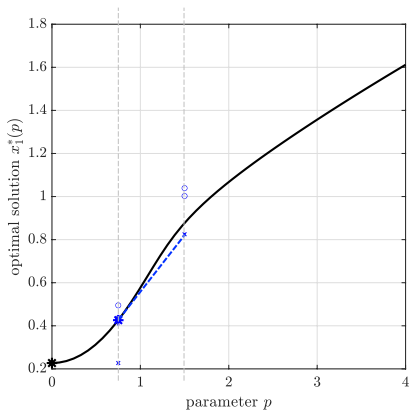
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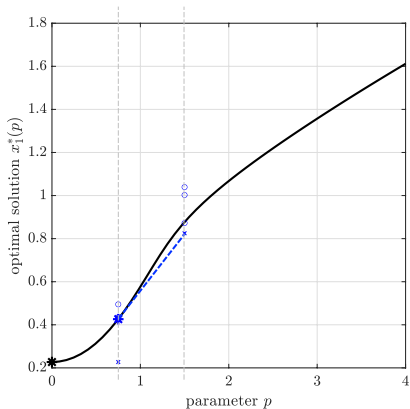
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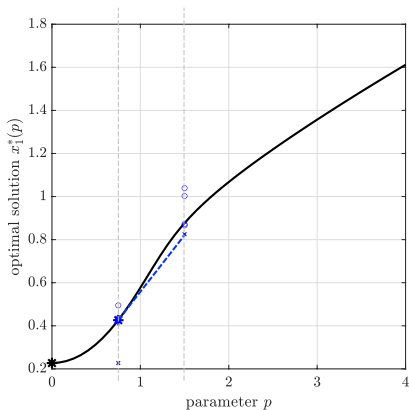
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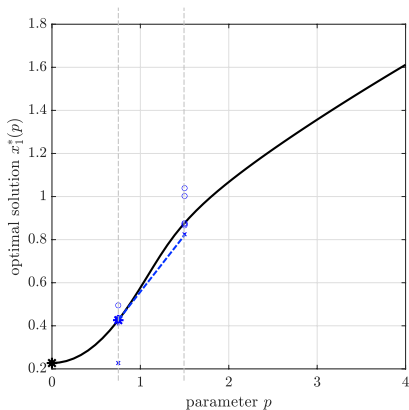
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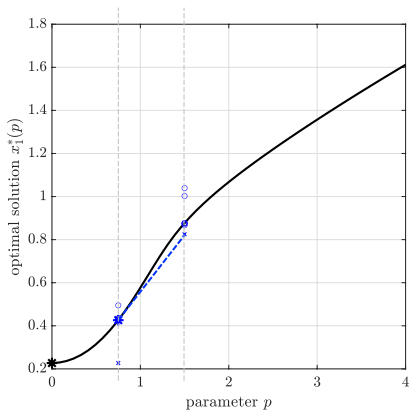
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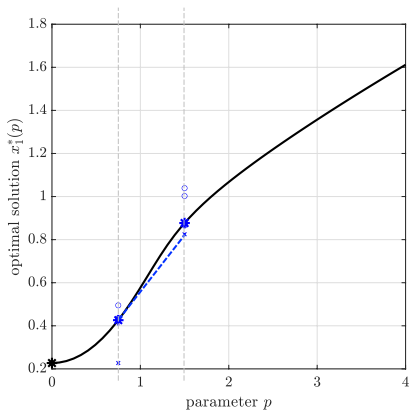
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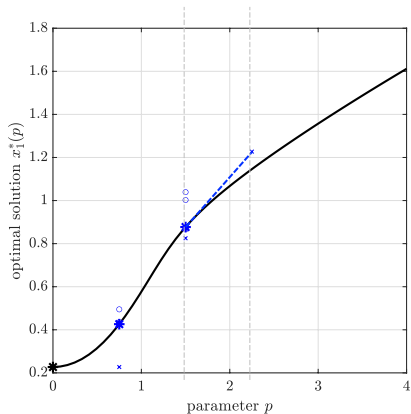
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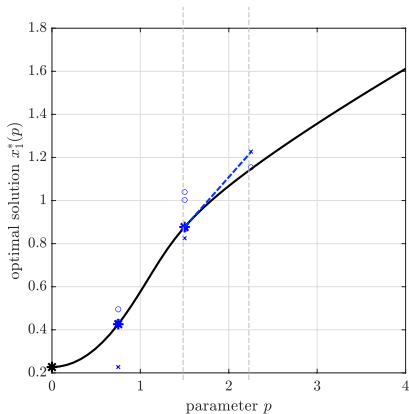
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Tangential predictor



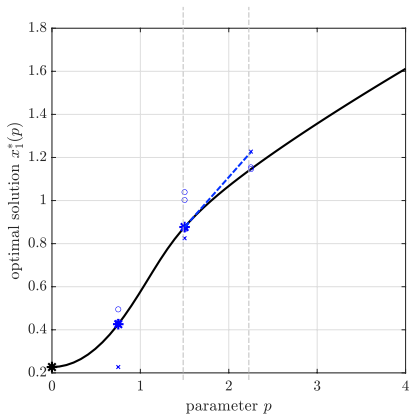
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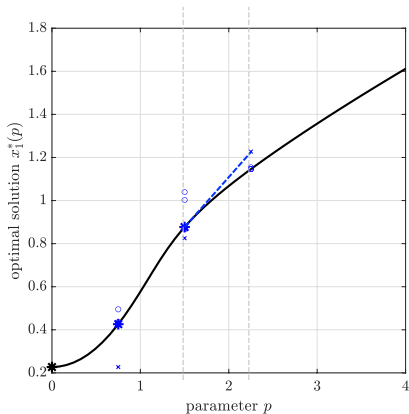
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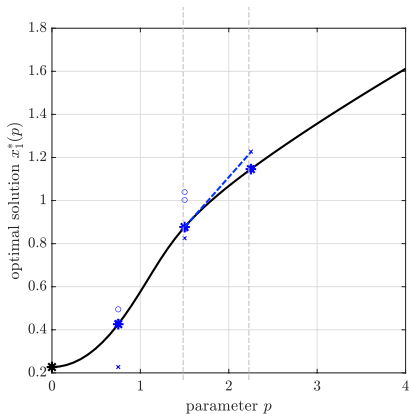
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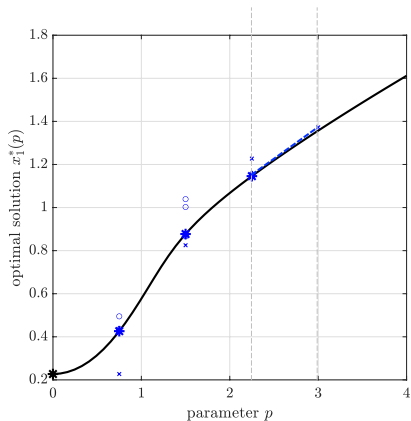
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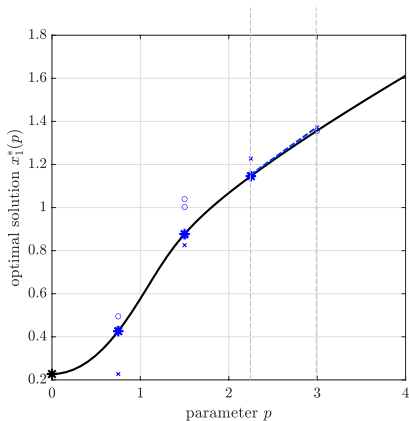
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Tangential predictor



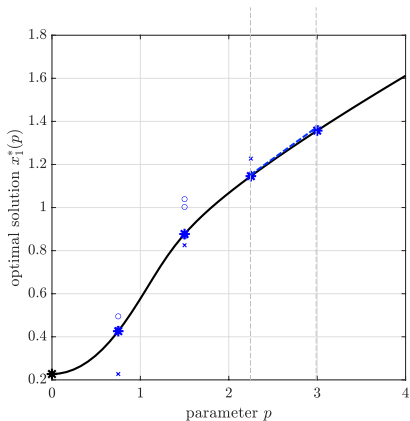
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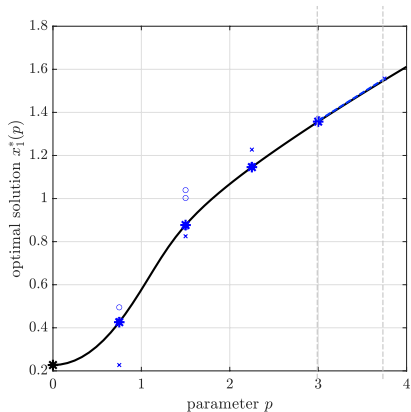
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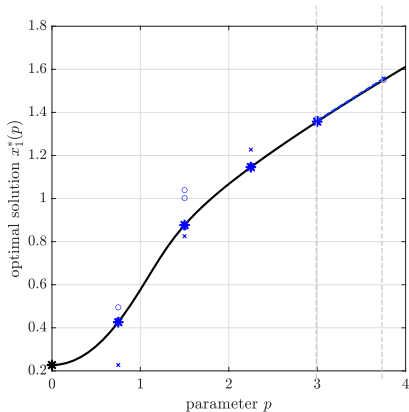
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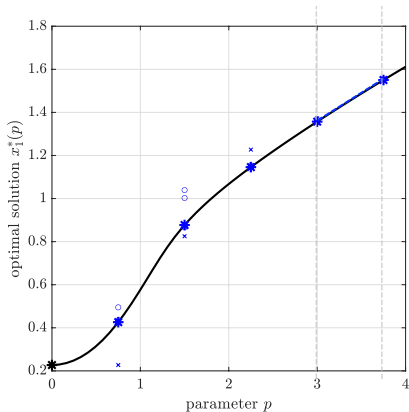
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Predictor corrector methods for path-following

QP Predictor

$$\begin{aligned} \min_{\Delta x} \quad & \frac{1}{2} \Delta x^\top H_k \Delta x + \Delta p^\top \nabla_{xp} \mathcal{L}_k \Delta x \\ \text{s.t.} \quad & \nabla_x g_k^\top \Delta x + \nabla_p g_k^\top \Delta p + g_k = 0 \\ & \nabla_x h_k^\top \Delta x + \nabla_p h_k^\top \Delta p + h_k \leq 0 \end{aligned}$$

SQP Corrector

$$\begin{aligned} \min_{\Delta x} \quad & \frac{1}{2} \Delta x^\top H_k \Delta x + \nabla_x f_k^\top \Delta x \\ \text{s.t.} \quad & \nabla_x g_k^\top \Delta x + g_k = 0 \\ & \nabla_x h_k^\top \Delta x + h_k \leq 0 \end{aligned}$$